Computer Mathemagical Merriment Spinning Onto Your Desktop

About this Issue's Programs and Graphics

This issue of **REC** was composed on a Macintosh Performa 450 with color monitor and printed on a GCC Technologies PLP II laser printer. The software used was *Microsoft Word 5.1a* for the Macintosh along with a variety of other programs and/or desk accessories, primarily for fonts and/or graphics, particularly screen dumps. Graphics of PC programs running on a '486 PC are courtesy of Inner Media's fast and excellent *Collage Complete for DOS and Windows*. Many clip-art graphics on the Mac come from MEI's *Shareware Breakthrough* CD --- or Dubl-Click's *Wet Paint* CD, or Zedcor's *Deskpaint* samples.

Note: For those running PCs instead of Macs I RECommend obtaining comparable ease of use and elemental desktop publishing by running *Windows* and *Microsoft Word for Windows*, respectively. Most programs in **REC** are for both PCs and Macintoshes. With adjustments, they may run on other platforms (whether supported or not by this publication).

Personal to Readers

Welcome to the latest double-issue of **REC**!

This issue heralds a number of minor firsts. For instance, mid-December to mid-January I was on America Online (AOL) almost every day. Several of our contributing editors, notably Harry J. Smith (on AOL) and John J. Straub (on Compuserve) have been deluging me with great submissions, ideas, programs, solutions, and correspondence. I also get many great graphic programs from Mal Lichtenstein. Keeping up with them is a real job!

In addition, I have not only been downloading interesting programs written by others but did upload a few of my programs and rehashes of past **REC** articles. The idea is to attract prospective *RECkers*. Results are modest at best. (In fact, they have been fruitless.) But who knows what effort will bring, particularly if you readers would help in spreading the word of **REC**!

This issue I welcome and thank our newest lifetime member-subscriber, Dr. James McElhatton of Malta. Jim has been living and working in Athens, Greece, and I am delighted with his enthusiastic support and response. Indeed, it goes beyond life membership, and at least he knows how much I thank him. But how much he reached out is suggested by the fact that Jim called me from Greece and Tunisia almost every day during the week of the west-coast earthquake and the east-coast blizzard.

On Jan. 21 northeast Pennsylvania had a new official record low temperature of -21 degrees Fahrenheit -- and other areas even claimed temps lower than that, some in the negative 30s.

Against that backdrop of calamities in the USA I welcome Jim to our most exclusive coterie!

Temperature Teaser

Discussing the temperature reminds me of this: I tell you that the temperature was x. You say: "Celsius or Fahrenheit?" I reply: "Doesn't matter." If I'm truthful and correct, what is the value of x?

Mac Hardware Upgrades

I finally succumbed to the temptation and bought myself a double-speed CD ROM drive for the Mac Performa 450 (the computer on which I compose **REC** and contact America Online). And, since I have been running out of room on this same Mac Performa's 120M hard drive, I'm also ordering a larger hard drive as a secondary storage medium.

Atari Support

Last time I told you about selling off an Apple IIgs (at a big profit), picking up an Apple IIc (at no net cost), and acquiring an Amiga 1000. This time I have sold off one of my older computers, a Tandy 1000, and used the money to buy a used Atari 1040 ST (got a terrific deal) *and* to pay for part of the cost of the new Mac CD drive, at least.

The sales allow me to keep solvent and purchase more equipment, which in turn enable me to support more of you in more ways. In turn I ask you all: Please send programs and disks for the new machines. I'll thank you, and so will my kids.

Editorial Board Shuffle

Lest I forget, I thank Steve Wagler, Dr. Henry Sonneborn, Maurice Servranckx, and Warut Roonguthai for their long past service on the Editorial Board. Membership on the Board is merely recognition of an unusually high level of **REC**-related activity, and I would not be surprised to see their names on the Board again. In any case,

coincidence or not, you will see all these names every issue anyway, as each is also a Life Member.

Contest News

I am pleased to report that my persistence has paid off with contest prizes. After my nagging Delrina Software to honor the commitment of prizes made by the former Amaze, Inc. (which Delrina acquired this past year), I received *five* copies of the terrific-looking *The Far Side Vol. III: Daily Planner and Calendar Publisher 3.0 for Windows*. There is also a Mac version, but all *four* winners (Dr. Bob Stong, Dr. Fred Galvin, Mr. Bill Mason, Mr. John Fowler) opted for the PC version, which I have personally sent to each. It promises to be most humorous. The cute cover shows a dinosaur looking at its daily schedule. Each day has just one "to do" item: *"Kill something and eat it"!*

This **REC** we have the results of the second contest for 1993 as well as a third, new one - a natural extension really - to bring in 1994.

Speaking of the new year, we have some late but welcome seasonal material from several sources.

Oh - and the *fifth* prize copy? *I* get to keep one -- the spoils of editorship and all that!

Enjoy REC #58-59 *Michael W. Ecker*

Important Communication Reminder:

- Include a disk (prefer PC or Mac) with your programs, letter, & text saved in ASCII format. *Please don't expect me to type in programs!*
- Send compiled, standalone .EXE files with the ASCII source (if using QuickBASIC, C, Pascal...).

America Online (aol): Please leave e-mail & PC / Mac programs for DrMichaelE. If you're not on AOL, use Internet address: DrMichaelE@aol.com

I remind you that I welcome your involvement and feedback for improvement to serve you better!

Answer to Temperature Teaser: x = -40. The linear relationship between Celsius/ Centigrade (C) and Fahrenheit is given by F = 1.8C + 32. (To derive this from scratch, use the point-slope equation of a line and the known boiling and freezing points for water.) We want to know when F = C. Solve simultaneously: C = 1.8C + 32, so .8C = -32, or C = -40.

TEMPERATURE SCALES
CELSIUS--FAHRENHEIT--KELVIN
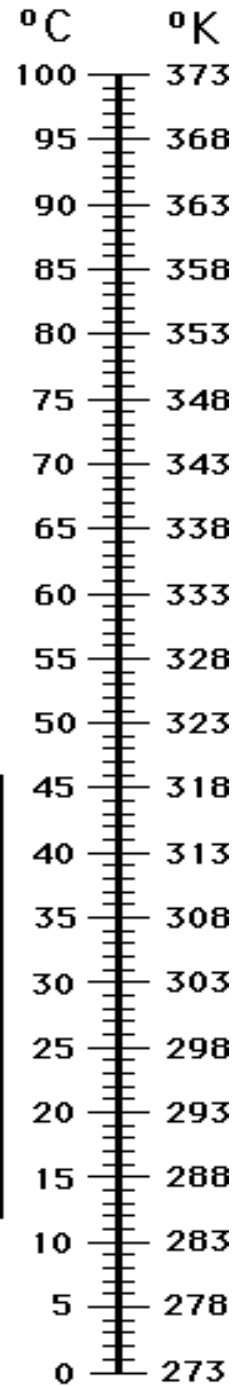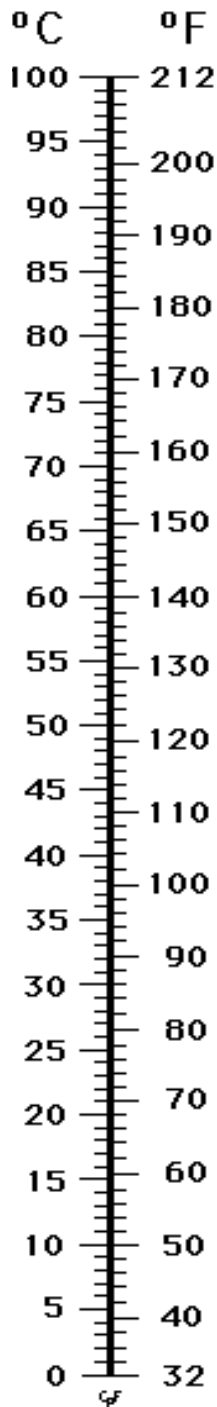
Temperature Conversion Formulas

$$°C = \tfrac{5}{9}(°F - 32)$$

$$°F = \tfrac{9}{5}(°C) + 32$$

$$°K = °C + 273$$

°C	°F
100	212
95	200
90	190
85	180
80	170
75	
70	160
65	150
60	140
55	130
50	120
45	110
40	100
35	90
30	
25	80
20	70
15	60
10	50
5	40
0	32

°C	°K
100	373
95	368
90	363
85	358
80	353
75	348
70	343
65	338
60	333
55	328
50	323
45	318
40	313
35	308
30	303
25	298
20	293
15	288
10	283
5	278
0	273

	°F	°C	°K
Steam Point	212.00	100.00	373.15
Ice Point	32.00	0.00	273.15
Absolute Zero	-459.69	-273.15	0.00
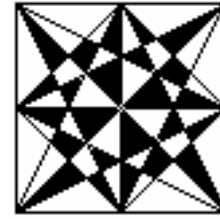
Correspondence: Letters, Solutions, Programs

Your letters, questions, solutions, programs, articles, corrections, and donations are most welcome. Write: *Dr. M. Ecker, REC Editor/ 909 Violet Terrace/ Clarks Summit, PA 18411 (USA)*. Include a self-addressed, stamped envelope if you need a reply. Or, contact DrMichaelE on America Online or DrMichaelE@aol.com via Internet.

Letters and Articles Policy - All materials sent to **REC** are subject to editing for grammar (e.g., the user inserts *his* disk, or *his or her* disk, but not *"their"* disk); length (as warranted by topic and treatment); and style. Perceived value and interest constitute the main consideration, with PC-compatibility or portability being important. I try to publish as many as feasible, but cannot print all. Those that I can print I often cannot print on a timely basis. *Publication consideration is improved by including a PC or Mac disk (any standard size, though I appreciate other disks: Atari 3.5", Amiga 3.5", Sanyo 5.25", TRS-80 Mod III 5.25", Apple II 5.25") with text & programs saved in ASCII* . *Label disks with name, address, disk type, contents*.

Note: More-detailed program & article guidelines are available free upon request. Please contact me.

Fractal Feather Friends?

In late November *RECker* Willem Kupper of The Netherlands wrote to inquire about different outputs corresponding to certain parameters in the feather fractals program we had Summer 1992. In turn I not only corresponded with him but also with Harry J. Smith and Dr. Clifford Pickover via e-mail in late December 1993. It was Cliff's work in the late *Algorithm*, after all, that inspired the original inclusion in **REC**. And Harry had so many feather fractal variants earlier that I thought he might also shed some light. (At the least, you readers should enjoy the correspondence, the program, and the picture, all of which I've edited.)

The question Willem asks is why the program runs properly in QBASIC but not in GWBASIC. My own suspicion - based mostly on narrowing down the issue using A=-.483 vs. -.484 - was that we are in another chaos situation of sensitivity to initial conditions. If we press <space> when we have the higher A-value(s) we keep getting the feathers, until finally we get three points. With the lower value(s) we keep getting just one point. Does this have to do with the famed theorem about period 3 implying chaos, for instance? Or, more likely, is this merely precision-related? Can any reader shed further light on this?

```
20 'Feather1.Bas - Pickover's Feather with new start, Last revised 1992-10-03
30 'Author: Harry J. Smith, 19628 Via Monte Dr., Saratoga CA 95070
40 'See Algorithm 3.3 July - Sept 1992, Page 10, by Dr. Clifford A. Pickover
50 'Also see REC June/July 1992 (Issue #48), Page 8, by Dr. Michael W. Ecker
60 'Modified slightly to add comments Dec. 1993 - Jan. 1994 by Dr. M. Ecker
100 SCREEN 9: 'REM out in Mac version
110 PRINT "Pickover's Fantastic feather fractals!"
120 PRINT "Space clears screen; x exits."
140 A = -.484: B = .93: : C = 2 - 2 * A
150 P = 9200000!: X = 1.001: Y = 0
160 W = A * X + C * X * X / (1 + X * X)
170 FOR N = 0 TO P
180 PSET (20 * X + 300, -20 * Y + 140) 'Scale/Flip/Center
190 Z = X: X = B * Y + W: U = X * X
200 W = A * X + C * U / (1 + U): Y = W - Z
```

```
210 K$ = INKEY$: IF K$ = " " THEN CLS
220 IF K$ = "x" OR K$ = "X" THEN END
230 NEXT N
240 'Conclude: A=-.484 okay, but not -.483
250 'Also, can erase earlier and still reproduce picture
260 'Until some point where degeneration is to 3 points (or 1).
```


It may be, indeed, more likely that the problem is processor-related rather than theoretical. Read on:

From: H.J. Smith -- Dear Mike,

If you want stability, you can change the initial X value from 1.001 to 1.01 and leave A = -.48, but the whole point of this feather was that it starts out looking like it has converged to a single point and then becomes a feather. Try running it at 8 MHz.

You would think that GWBASIC and QuickBasic would produce the same results since all the computations are being done in the coprocessor, but take line 200, for example:

200 W = A * X + C * U / (1 + U): Y = W - Z

If one of the BASICs is doing:

200 W = A * X + (C * U) / (1 + U): Y = W - Z

and the other is doing:

200 W = A * X + C * (U / (1 + U)): Y = W - Z
there could be a difference.

I tried putting in more parentheses to force them to be alike but could not get GWBASIC to work right with A = -.48.... One of the BASICs could be doing all the computations of a given statement in double-precision and converting the final results to single-precision. I tried putting in more intermediate steps

to prevent this, but couldn't get GWBASIC to work right with A = -.48 -- or QuickBasic to fail.

My answer is to change A = -.48 to A = -.484 and let it go at that.

The bottom line is that you cannot and should not expect two programs to produce the same results when they are evaluating the same variable assignments for numerically unstable equations.

Sincerely, *Harry*

The programs for PC and Macintosh appear on this issue's REC-on-Disk, along with a host of other programs and bonuses. REC-on-Disk is a disk supplement for REC subscribers with a PC or Macintosh. At least one disk comes with each issue of REC and does far more than save typing. Not the newsletter itself, it's a separate purchase: $29 annually in USA, $32 to Canada, $38 abroad.

Good Programming Uses of Lines and Circles

REC Contributing Editor John J. Straub illustrates how simple uses of the line and circle commands found in most BASICs can create stunning graphics and effects. First, we start with his X'mas tree, which he points out would have been better timed had he gotten it to me before *last* issue came out instead of Dec. 27.... Oh, well!

X'mas Tree
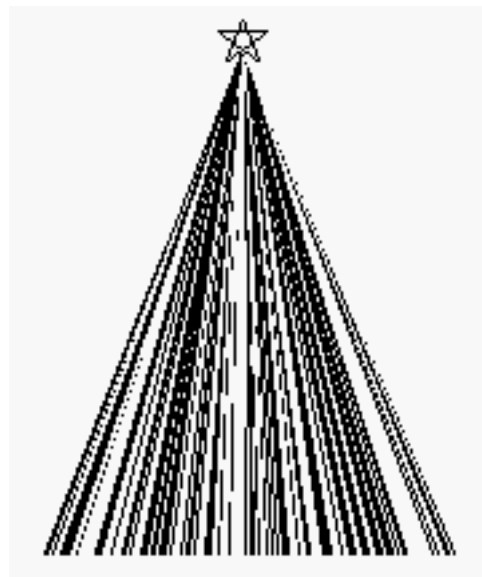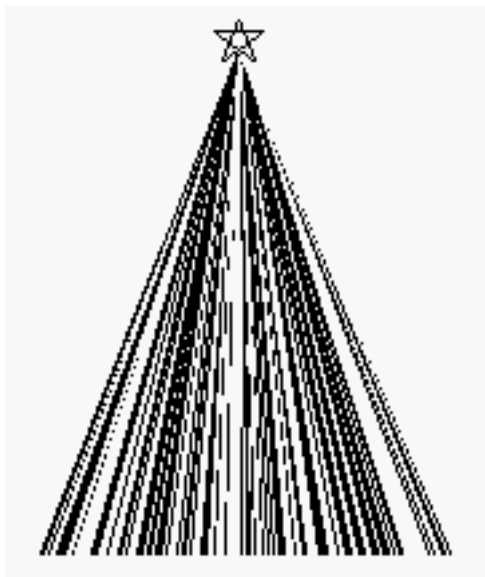

```
10 'Christmas Tree, PC version, John Straub
100 SCREEN 7: CLS: RANDOMIZE TIMER
110 X = 160: Y = 200
120  LINE (X, 0)-(X + 6, 15), 15
130  LINE -(X - 9, 4), 15
140  LINE -(X + 9, 4), 15
150  LINE -(X - 6, 15), 15
160  LINE -(X, 0), 15
```

```
170 L = INT(RND * 160) + 81
180 C = INT(RND * 15) + 1
190 S! = INT(RND * 32766) + 1
200 LINE (X, 12)-(L, Y), C, , S!
210 A$ = INKEY$: IF A$ <> "" THEN 230
220 GOTO 170
230 SCREEN 0: WIDTH 80: CLS : END
```

You can get some interesting effects making S! smaller. Don't make it bigger or you'll get an error in the next line.

Editor's Note: Experiment with different PC screen commands for different-sized trees. □ For the Mac version, keep the core above but make these minor esthetic modifications: Take out the screen command in line 100, add 5 to the y-coordinates in the line commands, delete 230, and take out parameter S! in line 200:

```
10 'Christmas Tree, Mac version, Dr. M. Ecker      150  LINE -(X - 6, 20), 15
120  LINE (X, 5)-(X + 6, 20), 15                    160  LINE -(X, 5), 15
130  LINE -(X - 9, 9), 15                           200 LINE (X, 17)-(L, Y+5), C
140  LINE -(X + 9, 9), 15                           210 A$ = INKEY$: IF A$ <> "" THEN CLS : END
```

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Dynamic Circles   Dynamic Circles   Dynamic Circles

```
10 SCREEN 7: CLS: 'John Straub, original PC version    50 CIRCLE (160, 99), 99, Y, X + .001, X
20 FOR X = -6.283 TO 0 STEP .01                        60 NEXT X
30 Y = INT(RND * 15) + 1                               70 IF LEN(INKEY$) THEN END
40 CIRCLE (160, 99), 99                                80 GOTO 20
```

Dynamic Circles
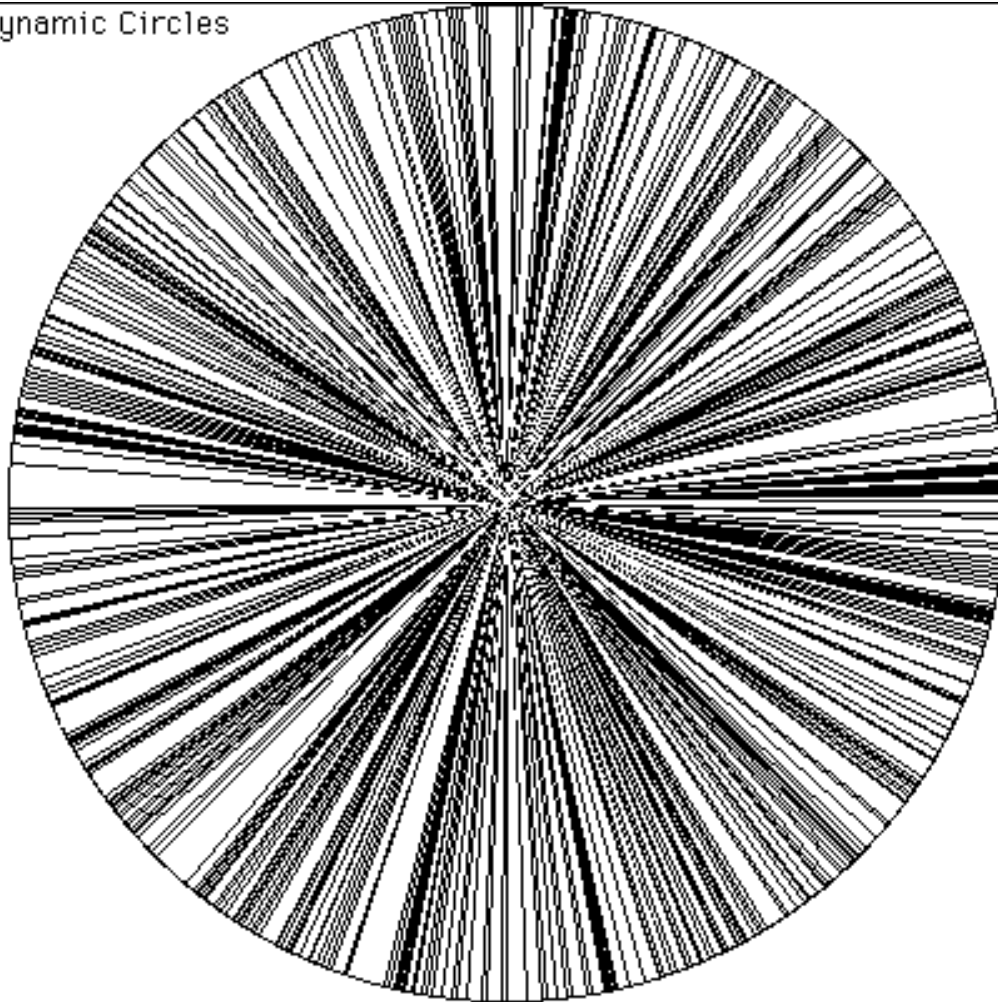
```
10 SCREEN 12: CLS: 'Dr. M. Ecker, PC alternate vers.    50 CIRCLE (320, 240), 210, Y, X + .001, X
20 FOR X = -6.283 TO 0 STEP .01                          60 NEXT X
30 Y = INT(RND * 15) + 1                                 70 IF LEN(INKEY$) THEN END
40 CIRCLE (320, 240), 210                                80 GOTO 20
```

```
10 CLS: 'Dr. M. Ecker, Mac version of Dynamic Circles        50 CIRCLE (300, 199), 199, Y, X + .001, X
20 FOR X = -6.283 TO 0 STEP .01                               60 NEXT X
30 Y = INT(RND * 15) + 1                                      70 IF LEN(INKEY$) THEN END
40 CIRCLE (300, 199), 199                                     80 GOTO 20
```

Triangular Fractals and Space Invaders from Mars
John J. Straub, Contributing Editor

John also sent a cute program he called Space Invaders. When I told him the output looked a lot like a fractal program, he told me it was an adaptation of an original fractal program - no author listed. So, first I present the original, followed by his own variation. I'll omit the pictures, variants, and Mac versions.

```
10 'This program will create a Fractal and then wait for you to press a key
20 'when it is complete. For a different Fractal REM out lines 150 and 160.
30 CLS
40 KEY OFF
50 SCREEN 9
60 LIMIT% = 200
70 MAXROWS% = 350
80 MAXCOLUMNS% = 640
90 FOR X1% = 1 TO MAXCOLUMNS%
100   FOR Y1% = 1 TO MAXROWS%
110     X = X1% / MAXCOLUMNS%
120     Y = Y1% / MAXROWS%
130     COUNT% = 0
140     WHILE X * X + Y * Y <= LIMIT%
150       IF Y <= .5 THEN 160 ELSE X = 2 * X: Y = 2 * Y - 1: GOTO 180
160       IF X <= .5 THEN 170 ELSE X = 2 * X - 1: Y = 2 * Y: GOTO 180
170       X = 2 * X: Y = 2 * Y
180       COUNT% = COUNT% + 1
190     WEND
200     PSET (X1%, Y1%), COUNT%
210   NEXT
220 NEXT
230 WHILE INKEY$ = ""
240 WEND
```

```
20 'Space Invaders, John Straub                100 WHILE X!^ 2 + Y!^ 2 < X!^Y!
30 DEFINT A-Z: SCREEN 8: CLS                   110 IF X! > .5 THEN X! = 2*X! - 1: Y! = 2*Y!: GOTO 140
40 XX = 640: YY = 300                          120 IF Y! > .5 THEN X! = 2*X!: Y! = 2*Y! - 1: GOTO 140
50 FOR J = 1 TO XX                             130 X! = X!*2: Y! = Y!* 2
60 FOR K = 1 TO YY                             140 Kolor = Kolor + 2
70 X! = J / XX                                 150 WEND
80 Y! = K / YY                                 160 PSET (J, K), Kolor
90 Kolor = 0                                   170 NEXT K, J
```

```
10 CLS                                        100 NEXT j
20 PRINT "Speckleband... Dr. M. Ecker"        110 NEXT k
30 PRINT : PRINT "Press ENTER..."             120 xr = 640 * RND: yr = 480 * RND
40 INPUT "", A$                               130 colr = 16 * RND
50 SCREEN 12                                   140 PSET (xr, yr), colr
60 FOR k = 0 TO 15                             150 'FOR dl = 1 TO 500: NEXT dl
70 COLOR k                                     199 IF INKEY$ <> "" THEN END
80 FOR j = 0 TO 29                             200 GOTO 120
90 LINE (0, 30 * k + j)-(639, 30 * k + j), k      (Use line 150 for time-delay in fast machines such as '486s)
```

Above program draws colored bands across the PC screen and then splashes tiny paint drops.

1) To use the time-delay in line 150 (e.g., for a '486), merely exclude the REMark apostrophe (').
2) Take out color & screen commands and program above will run in black & white on Macintoshes.
3) I'll skip the commercials about REC-on-Disk except to remind you this issue's programs are on it.

Percentage Power and Number Sense
Dr. M. Ecker, REC Editor

We live in a society that is rather innumerate (and illiterate - but that's another matter, and neither observation is news). We all know or meet people who boast of their ignorance of mathematics, which usually begins with fractions, decimals, and percents. Dependent on calculators / technology for what formerly was "mental math" (arithmetic) and living in a society that tolerates ignorance, they can get away with some shortcomings.

In the spirit of keeping arithmetic alive, let's look at percentage change. Even those with a doctorate in mathematics might make a new observation - at least I did. But first, here's an easy starter question:

A) Relatively speaking, which is worse?
• bubble gum going from one to three cents, *or*
• an automobile going from $4,000 to $5,000?

This is routine. Before we get to it, let's be fair and admit that some of the questions one can ask are more subtle. For instance, add these two teasers to the easier one above, but be careful:

B) You have a sale and temporarily reduce the price of a shirt by 20%. After the sale is over, by what percentage are you then raising the sale price of the shirt when you restore the pre-sale price?

C) On January 30th Managing Editor Ed Rogers of the Scranton Times - Tribune announced that the Scranton, PA newspaper was going to change its typeface to Nimrod and its print density from 8 lines per inch to 6.8 lines per inch. Mr. Rogers, who modestly included a gracious disclaimer about assuming his math was correct, stated that this change resulted in so-and-so percentage increase in type size.

I called him February 11th to question his claimed percentage increase in type size. Before I tell you what he had, what do *you* calculate it to be?

Primer: By definition, the *percentage change* or *relative change* in a quantity is the ratio of the change in the quantity to the original (i.e., earlier) value of the quantity. (The major difference in the two terms is that a percentage change is a relative change converted to a percent.)

Here, *change* refers to actual change or absolute change, defined as the latter value minus the former (even if the latter value is smaller, please note). The value taken as the former, original, or earlier (all of which I use synonymously) is dependent on the situation. A price may go up and then down, or vice-versa; "original" is taken as relative to that price movement only, not anything that happened still earlier or may happen later.

For instance, a hat goes up in price from $10 to $12. The actual change is +$2. The relative change is this

change over original price, or (12-10) / 10 = 2 / 10 = 1 / 5 = .2 = 20%, the latter being the percentage change. If the hat should go from $12 back down to $10, we then calculate (10-12) / 12 = -1 / 6 = -16 2/3%. Note: In the first situation the "original" price is $10; in the second, it is $12.

So, even though the actual change is the same in absolute value* - that is, $2 - the relative change depends not just on the change but also the original amount. The fact that the hat goes down by a smaller percentage reflects a smaller percentage of a larger "starting" value. This, of course, is the whole point.

(* The actual change is really not the same, either: +$2 in the first instance, and -$2 in the second.)

Now, some answers:

A) The bubble gum goes up (3-1) / 1 = 2 = 200%, while the car goes up (5000-4000) / 4000 = 1 / 4 = 25%. Though the gum may seem a frivolous item, the price inflation on it, relatively speaking, is far more serious than for the car.

B) Suppose the shirt were fancy and cost $100. During the 20% sale, it would cost $80, which we regard as the latter cost for the price reduction. However, for the calculation of the subsequent price increase from $80 to $100, $80 is the earlier or "former" price. We must raise the price $20, so change / earlier price = 20 / 80 = 1 / 4 = 25%.

Explanation: As with the hat, we must take a larger percentage of a smaller price (25% of $80 instead of 20% of $100) to achieve the same $20 change in absolute value.

Aside: For the purist who wishes to make the shirt's price x, independent of any specific value, go ahead and check the algebra. You'll still get the same percentages. *This illustrates a little trick*: When you need to calculate a ratio in a general question such as in B) - one that presumably doesn't depend on the particular values - pick a convenient value / values. [We use this in C) now.]

C) Now we come to the trickiest of them all. The typeface is going from 8.0 lines per inch to 6.8 lines per inch. Mr. Rogers at the *Times* was resourceful enough to place the change (1.2) over the original

amount (8.0) to get the ratio 1.2 / 8.0 = 3 / 20 = 15% in his feature.

But we come to what may seem to some a subtlety: *Does this measure what it purports to measure?*

In fact, 15% is incorrect. Note that the units used in getting 15% are not units of length, but linear density in lines per inch, and this is the source of this mistake. We must use units of length.

To do this correctly, let's use the simplifying assumption of picking a convenient value - say, one inch of type with which to begin. (We'll also ignore the issues of character widths and spacing between characters.) The original inch of type at 8 lines per inch occupied 8 lines. We now go to the larger typeface setting only 6.8 lines per inch. How many inches will be consumed by the same 8 lines now?

In the new typeface, each line occupies 1 / 6.8 of an inch. Since there are eight lines, we get 8 times 1 / 6.8, or 8 / 6.8 inches as the new larger size. The change in size is 8 / 6.8 - 1; the original length is 1. We obtain the ratio (8 / 6.8 - 1) / 1, which simplifies to 1.2 / 6.8 or 3 / 17 -- not the original 1.2 / 8.0 or 3 / 20 that the *Times'* managing editor must have had.

Inverse Proportions

For those who prefer a slightly more rigorous or theoretical/ algebraic approach, I actually started assuming k lines. Key is the *inverse proportion*: (number of lines per inch) times (number of inches) = k, or linear density x length = constant.

Thus, the original length was k / 8, the new length is k / 6.8, and so the relative change is given by:

$$\frac{(k / 6.8 - k / 8)}{k / 8}$$

This little "complex fraction" simplifies to 1.2 / 6.8 (independent of k), further simplifying to 3/17. Note that this correct ratio could be gotten by the seeming error in working with the wrong variable (density instead of length) and: a) calculating the density change in the reverse order (former minus latter instead of latter minus former) & b) putting that change over the *latter* instead of the former.

To put it simpler, you'll get the correct relative change for an original variable if you use an inversely related variable and invert the order. In this case, we invert the "8.0 to 6.8" to become "6.8 to 8.0" and calculate change over earlier = 1.2 / 6.8 = 3 / 17, which is correct!

My Mini-Theorem: Suppose quantities q and i are inversely proportional, so q*i = constant. To find the relative change in q given only values of i, one may calculate instead the relative change for i using i-values in the reverse (inverted) order.

Proof: Suppose quantity q goes from q_1 to q_2 in value, resulting in quantity i going from i_1 to i_2. (Of course, if q is increasing, i will be decreasing, if both q and i are positive-valued.) We have $i_1 q_1 = i_2 q_2$ (i.e., the product i*q is constant).

The relative change in q is given by $(q_2 - q_1) / q_1$, and we need to show this equals $(i_1 - i_2) / i_2$. But the first expression equals $q_2 / q_1 - 1 = i_1 / i_2 - 1 = (i_1 - i_2) / i_2$, as I claimed. ↕

Some other quantities inversely related (in theory):

- speed vs. time in traveling a fixed distance;

- pressure vs. volume of a gas (constant number of moles of gas at a constant temperature);
- number of workers needed to do a job vs. the time it takes them (assuming - unrealistically - that they work at a constant rate).

A simple "litmus test" to differentiate a direct proportion (ratio or quotient is constant) from an inverse proportion (product is constant) is to ask: If I double the first quantity, does this double the second quantity? Or does this halve the second quantity? In the first case, you have a direct proportion; in the second, an inverse proportion.

Calculus, too: Relative change in calculus uses differentials for local approximation of curves by line segments, but it is essentially the same thing. With a quantity Q, the approximate relative change in Q is dQ / Q, a dimensionless quantity in which dQ is the differential of the function Q.

Gems: All this about change reminds me of an old classic: If the earth were a (perfect) sphere and you increased the radius by just one meter, how much greater distance would be a walk around the earth (i.e., the circumference)?

Ed. Note: Allan's terrific magic squares have graced past issues of **REC** and I'm delighted to publish these.

On the following pages we also have another great contribution of magic squares, plus a program for generating much simpler squares... First, you should know that Ed Shineman creates his fantastic magic squares, regularly featured here in **REC**, without computer assistance! As for the program, I don't know the source. Either some *RECker* sent it to me (and the program became divorced from disks and notes), or, more likely, I got it off America Online. In any case, it appears to be satisfactory for generating smaller squares of odd order.

Mike's Magic Square
A 16x16 Square for 1994
Dedicated to REC's Editor
By Ed Shineman, New York, NY

Mr. Shineman created this magic square for 1994 on New Year's Day, 1994. The sum of the elements in any row, column, or full diagonal is 2056. Note the numerals that Ed emphasized (which I've bolded, underlined, and enlarged): the **94** in the upper-left corner (for the year) and the ones with my initials: **ME**. The total of the cells in each of the **M** and the **E** is 4112, which is double the square's magic sum, 2056.

94	195	62	211	46	3	254	19	238	115	142	99	158	179	78	163
164	61	196	45	212	253	4	237	20	141	116	157	100	77	180	93
91	198	59	**214**	**43**	6	251	22	235	118	139	**102**	**155**	182	75	166
165	60	197	**44**	**213**	**252**	**5**	236	21	**140**	**117**	**156**	**101**	76	181	92
89	200	57	**216**	**41**	8	249	**24**	**233**	120	137	**104**	**153**	184	73	168
167	58	199	**42**	**215**	250	7	**234**	**23**	138	119	**154**	**103**	74	183	90
87	202	55	**218**	**39**	10	247	26	231	122	135	**106**	**151**	186	71	170
169	56	201	**40**	**217**	248	9	232	25	136	121	**152**	**105**	72	185	88
85	204	53	220	37	12	245	28	229	124	133	108	149	188	69	172
171	54	203	**38**	**219**	**246**	**11**	**230**	**27**	**134**	**123**	**150**	**107**	70	187	86
84	205	52	**221**	36	13	244	29	228	125	132	109	148	189	68	173
174	51	206	**35**	**222**	**243**	**14**	**227**	**30**	**131**	**126**	147	110	67	190	83
82	207	50	**223**	34	15	242	31	226	127	130	111	146	191	66	175
176	49	208	**33**	**224**	**241**	**16**	**225**	**32**	**129**	**128**	**145**	**112**	65	192	81
96	193	64	209	48	1	256	17	240	113	144	97	160	177	80	161
162	63	194	47	210	255	2	239	18	143	114	159	98	79	178	95

Magic Square Generator (PC & Mac & Others)

```
10 CLS : PRINT "Magic Square Generator"
20 INPUT "Enter an odd number (side):"; N
30 PRINT : DIM M(N, N)
40 K = 1: N1 = 1: I = 1
50 J = (N + 1) / 2
80 M(I, J) = N1: N1 = N1 + 1
90 IF N1 > N * N THEN 310
100 IF K < N THEN 150
110 K = 1: I = I + 1: GOTO 80
150 K = K + 1: I = I - 1: J = J + 1
180 IF I <> 0 THEN 200
190 I = N
200 IF J <= N THEN 80
250 J = 1: GOTO 80
310 FOR I = 1 TO N
320 FOR J = 1 TO N
325 IF M(I,J)<10 THEN PRINT " ";
330 PRINT M(I, J); " ";
340 NEXT J
350 PRINT : PRINT
360 NEXT I
370 IF INKEY$="" THEN 370
```

Comment: I added line 325 to align at least two-digit numbers a bit better on some machines; use it if and only if it helps. Line 370 prevents Macintosh BASICs from changing windows and thus erasing the screen at the end of the program. I did not try to study the program or to improve it in other ways. I leave this to you; your own programs are hereby solicited.

The programs for PC and Macintosh appear on this issue's REC-on-Disk, along with a host of other programs and bonuses. REC-on-Disk is a disk supplement for REC subscribers with a PC or Macintosh. At least one disk comes with each issue of REC and does far more than save typing. Not the newsletter itself, it's a separate purchase: $29 annually in USA, $32 to Canada, $38 abroad.

TIME TUNNEL
John Straub

Ed. Note: Around Christmas, John sent this nifty program. The illusion is one of moving perpetually through a tunnel. I modified to allow PC variations and Mac use, but won't show all here. (Here's a freeze-frame snapshot.)
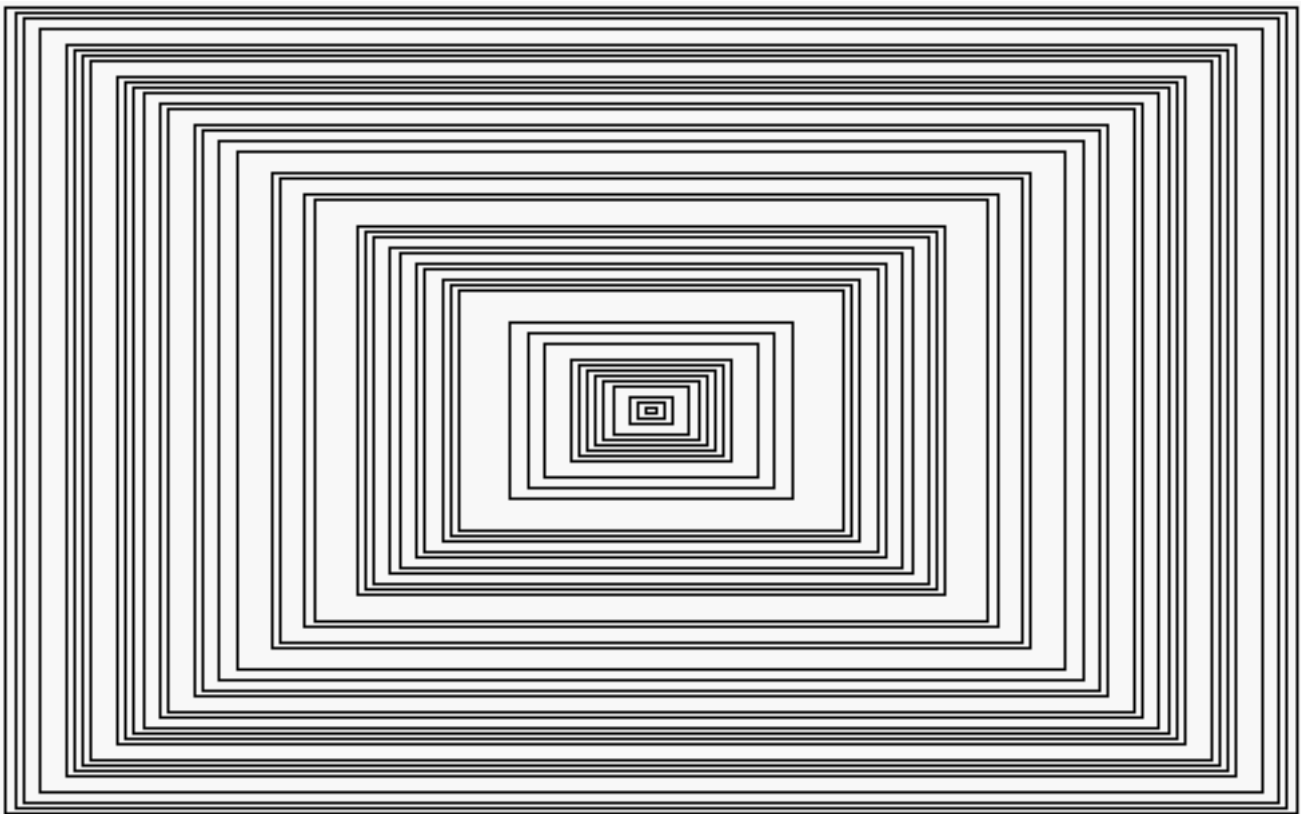

```
100 SCREEN 7: CLS: RANDOMIZE TIMER        160 LINE (XX + X, YY - Y)-(XX - X, YY + Y), C, B, S!
110 XX = 160: YY = 100: ' Use 300, 200 on Mac   170 A$ = INKEY$: IF A$ <> "" THEN 200
120 FOR Y = 1 TO 100 STEP 2               180 NEXT Y
130 X = Y * 1.6                           190 GOTO 120
140 C = INT(RND * 15) + 1                 200 SCREEN 0: WIDTH 80: CLS: END
150 S! = INT(RND * 32766) + 1             (Mac: Omit 150, S! in 160, all Screen, Color, Width...)
```

Note: To avoid repetition, I'll omit other variations of commands, screens, etc. on these. Go ahead and experiment on your own!

RAY GUN
John Straub

Editor's Note: As does John's *Tunnel* program, *Ray Gun* makes good use of simple line commands.

```
'Ray Gun, PC version by John J. Straub          'Ray Gun, Mac modification by REC's Editor
SCREEN 12, 0                                     CLS
DO                                               WHILE INKEY$=""
X = RND * 400                                    X = RND * 400
Y = RND * 200                                    Y = RND * 200
LINE (0, 240)-(X, Y + 150), RND * 16             LINE (0, 240)-(X, Y + 150)
LINE (0, 240)-(X, Y + 150), 0                    LINE (0, 240)-(X, Y + 150), 0
LOOP                                             WEND
```

Zeros of Functions by Interval Bisection
Dr. M. Ecker, REC Editor

To solve equations in one variable for which nice analytical techniques may be lacking, one can transpose terms to obtain an equation of form $f(x) = 0$. Then one can usually apply Newton's method or interval bisection. The idea for the latter is to guess an interval $[a, b]$ on which f has a zero. We need to have $f(a)$ and $f(b)$ with different signs. Then we bisect the interval. At the midpoint we consider the sign of the function's value. Iterate as needed.

For instance, to solve a certain Calculus problem last semester, I was led to setting $4x^3 - 6x - 12 = 0$ and solving. (It entailed a fairly standard minimization of a function, requiring setting up the function and then setting its derivative to zero. The problem was fun, but it was also fun to write a program.) We've had Newton's method and interval bisection in **REC** before, but I had not previously written my own for interval bisection. This one works well on both PCs and Macs:

```
10 CLS
20 PRINT "Function zeros by interval bisection (for continuous functions)"
30 PRINT "Dr. Mike Ecker, REC, 1993": PRINT
40 DEF FNf(x) = 4*x^3 - 6*x - 12
50 PRINT "Please give me an interval in which to start searching"
60 PRINT "Specifically, specify a and b of [a,b] such that the function"
70 PRINT "in line 40 takes on values with opposite signs at a and b."
80 INPUT "a, b with a < b"; a, b: IF a>=b THEN PRINT "a must be < b!": GOTO 80
85 PRINT a; FNf(a), b; FNf(b)
90 IF ABS(FNf(a))<.00001 THEN PRINT "We're done! Use x ="; a: GOTO 200
100 IF ABS(FNf(b))<.00001 THEN PRINT "We're done! Use x="; b: GOTO 200
110 IF FNf(a)*FNf(b)>0 THEN PRINT "No good... Same sign...": GOTO 50
120 x=(a+b)/2
130 IF FNf(x)*FNf(a)<0 THEN b=x: GOTO 85
140 IF FNf(x)*FNf(b)<0 THEN a=x: GOTO 85
150 PRINT "Solution is x ="; x
200 IF INKEY$="" THEN 200: REM END on Mac when ready
```


The programs for PC and Macintosh appear on this issue's REC-on-Disk, along with a host of other programs and bonuses. REC-on-Disk is a disk supplement for REC subscribers with a PC or Macintosh. At least one disk comes with each issue of REC and does far more than save typing. Not the newsletter itself, it's a separate purchase: $29 annually in USA, $32 to Canada, $38 abroad.

Bell Numbers, Catalan Numbers, & The 15 Ways of Factoring 210

Gary W. Adamson
P.O. Box 124571
San Diego, CA 92112

Abstract: Bell numbers (1, 2, 5, 15, 52, ...) denote the number of ways of factoring a number with distinct prime factors.

Shown are the 15 ways of factoring 210 = 2 x 3 x 5 x 7 (or *any* number with exactly four distinct prime factors).

The Catalan numbers can be derived from the Bell numbers using this geometric depiction of factoring. Just count the number of "crossovers" in the corresponding picture and subtract from the Bell number.

At right, we have only one crossover, namely the figure for 10x21. So, we can subtract 1 from the Bell number 15 to get the Catalan number 14.

Catalan numbers can also be derived from Pascal's triangle -- but that's another story.

Dr. Jeffrey Shallit (University of Wisconsin, Madison) is credited with this algorithm for Bell numbers: Start with 1; see figure below. In successive rows, repeat the rightmost term of the previous row for the first term of the row. For the other terms of the new row, add term to the left and term above that.

```
 1
 1   2
 2   3   5
 5   7  10  15
15  20  27  37  52
```

Reference: Martin Gardner's May 1978 *Scientific American* column.

6x5x7

21x2x5

10x21

15x2x7

30x7

6x35

35x2x3

70x3

14x15

14x3x5

42x5

210

10x3x7

105x2

2x3x5x7
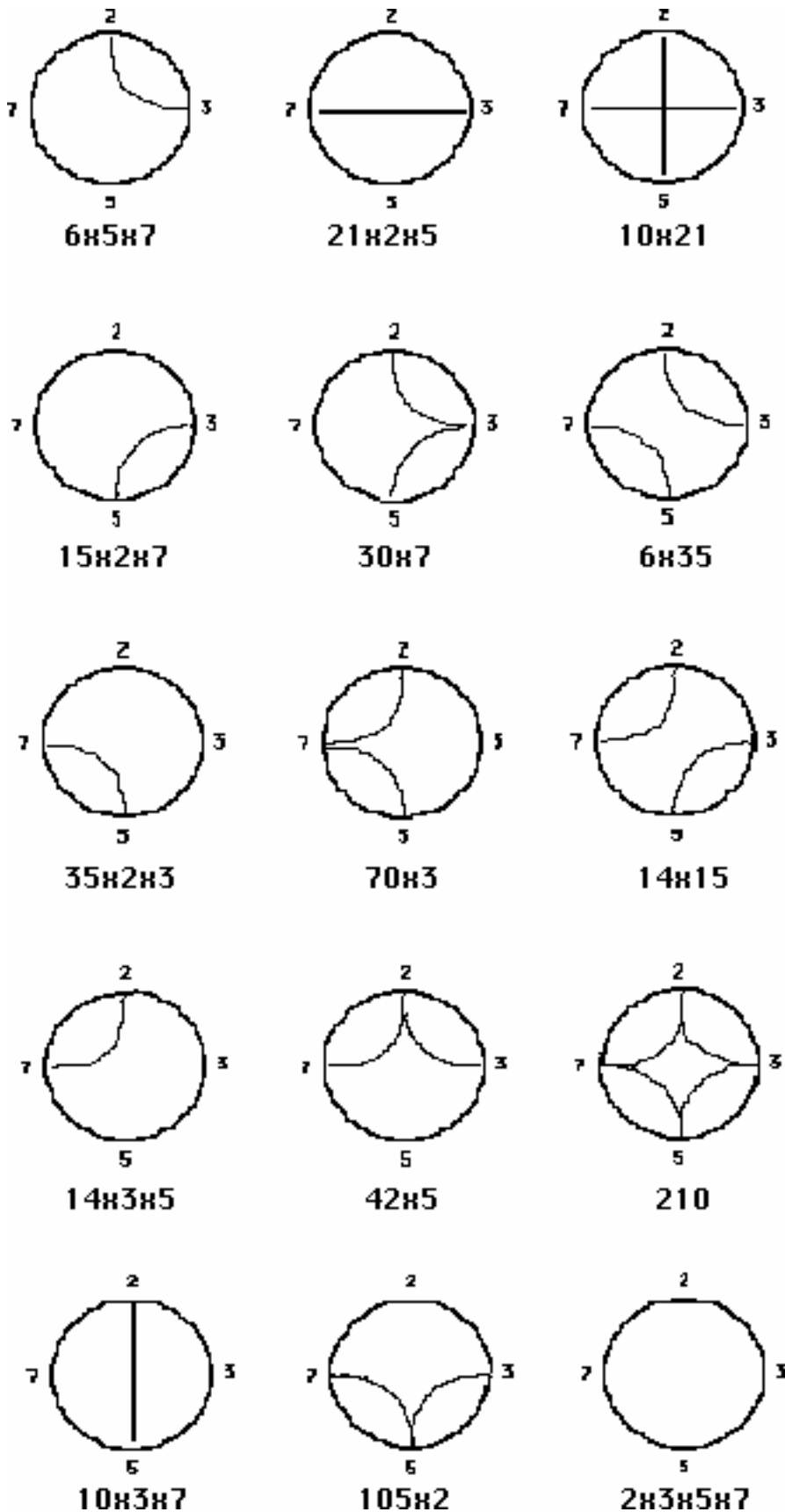
Explosion, by Mal Lichtenstein
Special to Dr. M. Ecker's REC, (c) 1994
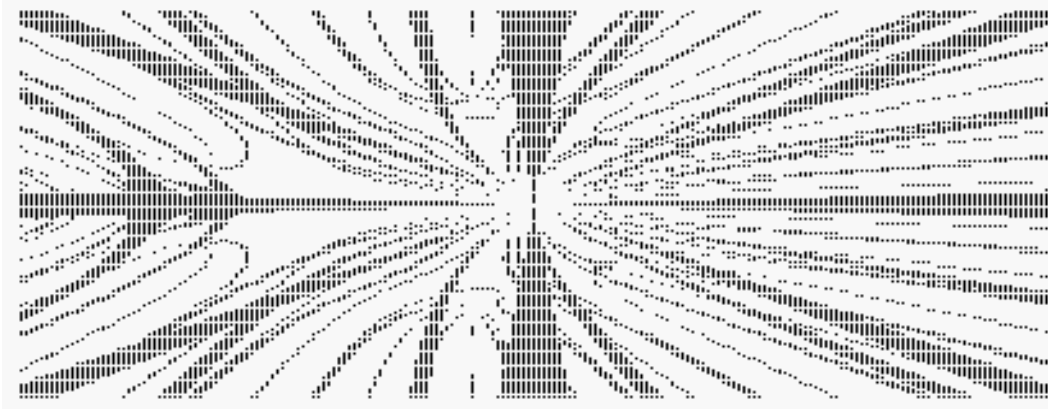
Contributing Editor Malcolm Lichtenstein sent in this program (code below, picture above) in December. I modified it to add little at first beyond the needed screen command for a PC with VGA color and QBASIC or QuickBASIC. The first result on a PC comes out in blue and green. It runs almost as is on other platforms (e.g., Macintosh BASICs).

I spent some fun hours with successive modifications to Explode1 (Explode2, Explode3, etc.) to provide varying effects, including portion of screen covered, number of colors, and other factors. I show here most of the changes I explored.

After typing in each variation, save each program with a different file name and then make only the changes shown. (REC-on-Disk subscribers, of course, need not type, and they get *all* source code modifications on disk, including ones not shown here.) Pictured above is Explode2.bas on my Macintosh Performa. The next page shows one of the later modifications on my '486 PC. The picture is more stunning on screen; you have to see the colors yourself! (The PC screen dump next page comes courtesy of the fast, excellent, and highly RECommended Collage Complete for DOS and Windows, by Inner Media.)

Explode1.bas
```
CLS : SCREEN 12: ' Screen added for PCs only
PRINT "    Explosion, by Mal Lichtenstein"
PRINT "    Special to Dr. M. Ecker's REC, (c) 1994"
FOR cx = -2 TO 2 STEP .04
FOR cy = -2 TO 2 STEP .05
x = 0: y = 0
FOR n = 1 TO 5
xx = x * x - y * y + cx
yy = 2 * x * y + cy
sx = ABS(xx - x) + .001
sy = ABS(yy - y) + .001
sqx = ATN(sy / sx)
sqy = ATN(sx / sy)
e = 1.5
IF sqx > e OR sqy > e THEN col = 1: GOTO escape
x = xx: y = yy
NEXT n
col = 2
escape:
PSET (100 * (cx + 2), 40 * (cy + 2)), col
NEXT cy
NEXT cx
100 IF INKEY$ = "" THEN 100
```

Explod2.bas (Make just changes shown.)
```
FOR cx = -2 TO 2 STEP .02
FOR cy = -2 TO 2 STEP .03
PSET (160 * (cx + 2), 100 * (cy + 2) + 50), col
```

Explode3.bas
```
sx = ABS(xx - x) + .001
sy = ABS(yy - y) + .001
IF sqx > e OR sqy > e THEN GOTO escape
```
Also: Delete COL = 2.
*Change col in PSET command to 20 * sqx * sqy.*

Explode4.bas
```
FOR cx = -2 TO 2 STEP .0075
FOR cy = -2 TO 2 STEP .01
```

Explode5.bas
```
FOR cx = -2 TO 2 STEP .00625
FOR cy = -2 TO 2 STEP .009
```

Explode6.bas
Change 20 to 10 in the PSET command.

Explode7.bas
```
FOR n = 1 TO 7
```
Also: Change 10 to 7.2 in the PSET command.

Youmans' Conjecture
Dr. Charles V. Youmans (Versailles, Kentucky), and edited by Dr. M. Ecker

Editor's Note: Charles was a REC subscriber for several years, until two months ago. He did not renew, citing an alleged lack of mathematics in these pages. Since I occasionally get some non-renewals citing too much math, I conclude that I'm probably just about right for my mission and target audience.

Now that Charles is no longer a RECker, somehow this has translated to his sending in material. Go figure...but do read on:

Despite the hoopla surrounding the proof of Fermat's Last Theorem, there remain many other unproven mathematical conjectures concerning an infinity of numbers. Such unresolved conjectures have neither been proven by general analysis nor disproven by explicit counterexample.

(*Philosophical Aside by Editor:* It is easier to destroy than to create. Thus, no matter how many examples are found as evidence for some conjecture involving an infinity of numbers, generally they cannot suffice to establish it. However, a single counterexample is sufficient to disprove a generality. Such is logic. In real life, however, we have an analogy insofar as it is easier to destroy than create. Never mind the deterioration of language and civil society; one may merely consider the relative ease vs. difficulty in destroying vs. creating a pane of glass... Try both and you'll see what I mean. But Gresham's Law [bad drives out good] aside, do note one distinction that *does* exist with some conjectures: They may be intractable enough to defy finding a counterexample, too. Or, as you'll see in Youmans', it can be just as hard to prove the conjecture wrong, because you're trying to prove the finiteness of a set rather than find an example.)

Goldbach's conjecture is that every even number greater than 2 is the sum of two prime numbers. Over the years this has been verified for more and more numbers. The advent and eventual widescale use of computers have served to accelerate the testing of more cases. Here's my new conjecture:

Take the prime numbers (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...) one at a time. For each prime p, form p* by taking the digits of p-1 and adjoining/ concatenating the digits of p. For instance, for p = 13, p-1 = 12, so we form 13* = 1213, again a prime. Does this happen infinitely often? That is, is p* itself prime for infinitely many primes p?

Here are the results using just the first few primes:

- 2* = 12 (non-prime)
- 3* = 23 (prime)
- 5* = 45 (non-prime)
- 7* = 67 (prime)
- 11* = 1011 (= 3 x 337, non-prime)....

Youmans' Conjecture: There are infinitely many primes p for which p* is also prime.

Editor's Further Comments: If the result is true, can we find an asymptotic density of such primes p? You might consider a program to generate primes p in order, calculate the corresponding values p* and test for primality, keep track of the number of each, and periodically report the fraction of values of p (up to the current last one tested) for which p* is also prime.

Note that I have also edited to provoke the tantalizing question of whether there is a pattern of alternation of composites (non-primes) and primes. Is there? If so, why? If not, is there any other pattern? What can we say?

Easy Math Problems for 1994

Thanks to Titu Andreescu (Illinois Mathematics & Science Academy) and Feb. 1994 *Illinois Mathematics Teacher* for these.

Find the smallest 19-digit number whose digits add up to 94, and the largest 94-digit number whose digits add up to 19.

Find all positive integers n such that n + 19 divides n + 94. (I.e., n + 94 is an integer multiple of n + 19.)

Determine all triples of prime numbers (p, q, r) such that p + 2q + 6r = 1994.

Find the last three digits of 19^{94}. (Ed. Note: We've done such in the past, but I've opted to include this.)

The only solutions for n+19 dividing n+94 are n=6 (25 divides 100) and n=56 (75 divides 150). To derive these and thus also prove these are the only solutions, we have n+94 = k(n+19) = kn + 19k for some positive integer k. So, (k-1)n = 94 - 19k, forcing k ∈ {1,2,3,4}. (I.e., k<5 because 94 - 19k is nonnegative.) k=1 leads to nonsense, and k=3 leads to a nonintegral value of n. If k=2, then n=94-38=56, and if k=4, then 3n=94-76=18, so n=6.

For p + 2q + 6r = 1994, note that 2q, 6r, and 1994 are each even, so p must be, too. Thus, p=2, the only even prime. Then q + 3r = 996. If either of q or r is 2, you can verify that there is no solution, so each is odd. In fact, q must be 3, because 3r and 996 are each multiples of 3, and so q is, too, with the primality of q forcing it to be 3 itself. Finally, r=331, a prime. We conclude that the only solution is (p, q, r) = (2, 3, 331).

Great Graphics: "Sin Tema"

Ariel Arbiser, Dept. of Computer Science
University of Buenos Aires, Argentina

```
'Ariel Arbiser, sin tema program 1
'PC QuickBASIC, EGA
DEFINT I
CONST xl = -3, xr = 3, yl = -3, yr = 3
SCREEN 9
WINDOW (xl, yl)-(xr, yr)
CLS
c = 0
FOR y1 = -13 TO 13 STEP .39
c = c MOD 15 + 1
FOR x1 = -13 TO 50 STEP .39
x = x1
y = y1
FOR I = 1 TO 62
x = x / (ABS(y) + 1) + y / 4
y = y / (ABS(x) + 1) - x / 2
PSET (x, y), c
NEXT I
NEXT x1
NEXT y1
```

```
'Ariel Arbiser, sin tema program 5
DEFINT I, T
CONST xl = -3, xr = 3, yl = -3, yr = 3, xs = .05, ys = .06
SCREEN 9
WINDOW (xl, yl)-(xr, yr)
CLS
c = 9
FOR y1 = yl TO yr STEP ys
FOR x1 = xl TO 3 * xr STEP xs
x = x1
y = y1
FOR I = 1 TO 10
xn = x / (ABS(y) + 1) + y / 4
y = y / (ABS(x) + 1) - x / 4
x = xn
PSET (x, y), c
NEXT I
NEXT x1
c = c + .1
NEXT y1
```

Editor's Comments: Prof. Arbiser provided these two to me over America Online - plus three others that I'll include on REC-on-Disk and/or the next Farrago. The b/w screen dumps of Ariel's color graphics (one is on next page) don't do them justice; they're colorful, swirling galaxies on screen! I have not converted to other BASICs but it should not be too difficult. For instance, for GWBASIC I would remove the CONST declaration and simply treat the rest of the line as separate assignments (with separate lines for each or with colon as delimiter between assignments). Mac users should also remove the Screen command, which the Macintosh doesn't need/ use. (I won't even touch the Window command or the adaptation of parameters for use in low-resolution systems such as TRS-80 and Apple II lines; I'll be glad to receive *your* disks showing *your* efforts.)

Incidentally, I heartily re-RECommend Inner Media's *Collage Complete for DOS and Windows*. With it I took the next page's screen shot on my '486 PC easily and then printed it out afterwards in about 10 seconds on my HP LaserJet II. I did not appreciate how good this product (Collage Complete) is/ was until Bill Dorion told me of waiting times on the order of 15-20 minutes for printouts using such competitive products as Pizazz Plus (which I have never solicited, owned, or even tried).

Prime Prize Contests

Mission Recap: Given the digits 0 through 9, find two or more prime numbers that collectively use the 10 digits each exactly once and such that the product of your numbers is maximal.

Most solvers agreed on the answer, though we caught a few who *assumed* (incorrectly) that the largest product would have to consist of two five-digit primes.

For want of room I am printing just one letter now. As room allows in future issues of **REC** we may have other letters and programs that shed additional light or insight.

From: Donn Kirk (Pleasant Hill, Oregon - 1/94)

Dear Mike:

For "Prime Prize Contest":

Maximum product using two primes:

9643 x 875201 = 8,439,563,243

... using two five-digit primes:
96401 x 87523 = 8,437,304,723
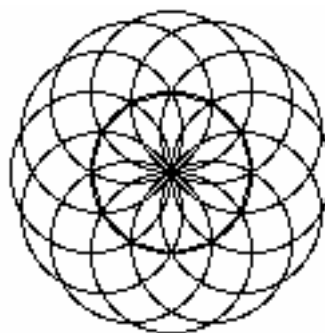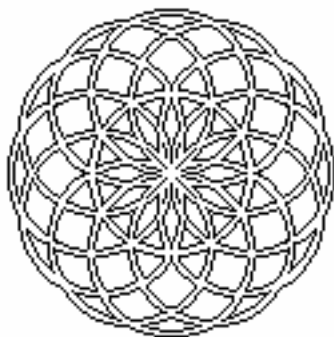
... prime times composite:
96431 x 87520 = 8,439,641,120

New Mission/ Contest: Same as before, but the product of your primes (using 0-9) is to be *minimal*. Send your entries (letters, disks of text and/or programs, etc.) to my address or via America Online.

CLOSE ISSUE #58-59

Because of the time-frame (it's 1994 Feb. 21), I am going to postpone intended features once more and forget my plan of a 30-page issue. Next month I would like to devote **REC** to catching up.

Before closing, I pass on the news of the death of Robert High, a **REC** contributing mathematical editor and longtime problem-solver. According to a notice in the latest *Journal of Recreational Mathematics*, Bob died while hiking in his beloved Chile almost 14 months ago (1993 January 8).

Graphic Games Galore Update: During January I updated this PC collection of a dozen great programs not only to

add Charles Kluepfel's terrific *Sorry!* program (requires VGA, mouse) but to add a routine for hard-drive installation and a simple menu system. Other programs include *MineSweeper*, two great *MasterMind* programs, *Jotto*, two *Slot Machines*, *4-in-a-Row*, *Super-Blackjack (Game of 110)*, *Monopoly*, *Joystick Saucer Invasion*, *Match-It*, etc. *RECkers* who have renewed at least through 1994 *and* who previously purchased an earlier version may have a free upgrade upon request to me. Other *RECkers* who renew for Vol. 9 (Apr. '94 - Mar. '95) may have the $28 collection for $19 until 1994 April 15. New life subscribers get the collection as a freebie. All other *RECkers* may order for $28 (includes shipping); the price to non-subscribers will be $39.

Upcoming: More great fractals/ graphics; math teasers, puzzles, & programs; plus reviews, CD trends, offerings; reader solutions/ corrections/ improvements; and backlog/ updates: N-Persistence, Finding the Next Number, Beatty Sequences, Why N-1 Cusps in Circle Residue Designs; Martin Gardner, older corrections, etc. Lots to catch up on next month... *Have you renewed your subscription yet?!*

Until then - Bye!   Mike